Conservation of Momentum:

"*In a closed system, the total momentum of the system remaines unchanged by anything which takes place within the system.*" [Discovered by Huygens]

total momentum before the collision = total momentum after collision

$$\mathbf{m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}}$$

The momentum before or after a collision is the same. [momentum is "conserved"]

The loss of momentum of one body = gain in momentum in another body. $\mathbf{[-m_1\Delta v = m_2\Delta v]}$

As an example, when a golf club head hits a ball the increase in (v) of ball causes a decrease in (v) of club. Since (v) is changing (a) is changing. The force of club on ball (F_1) equals force of ball on club ($-F_2$). [law #3] Thus, by combining the 2nd law with the 3rd law, we find that;

$F_1 = -F_2 = m_1a_1 = -m_2a_2 = {}^{m_1(v_f - v_i)}/_{t} = -{}^{m_2(v_f - v_i)}/_{t}$ since (t) is the same,

$Ft = m_1(v_f - v_i) = -m_2(v_f - v_i)$ *or* $\mathbf{m_1\Delta v = -m_2\Delta v}$

$m_1v_{1f} - m_1v_{1i} = -m_2v_{2f} + m_2v_{2i}$ *or* $\mathbf{m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i}}$

ex; A 900 g handgun fires a 8.5 g bullet at a velocity of 375 m/s. What is the recoil velocity (v_{1f}) of this gun?

sol; $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

Since the initial velocities are zero -----> $0 = m_1v_{1f} + m_2v_{2f}$

Solving for v_{1f} ------> $v_{1f} = {}^{m_2v_{f2}}/_{m_1} = {}^{(8.5\text{ g})(375\text{ m/s})}/_{(900\text{ g})} = \mathbf{3.5\ m/s}$

ex; A father (m_1 = 70 kg) and daugther (m_2 = 30 kg) are together and at rest on an ice rink. They "push off" each other, and he moves off with a velocity of 2 m/s. What is her velocity (v_f)? [Neglect friction.]

sol; Since $v_i = 0$, $m_1v_{1f} + (-m_2v_{2f}) = 0$ Solving for $v_{2f} = {}^{(70\text{ kg})(-2\text{ m/s})}/_{30\text{ kg}} = \mathbf{-4.7\ m/s}$

Although the system (father & daughter) had zero momentum at first, the total momentum of the system in the end is the same. Remember momentum is a vector quantity with a +/- direction assigned. Therefore, with one moving in a positive direction, and the other in a negative direction, the momentum of the system as a whole remains unchanged.

ex; A 1.2 kg wood block (m_1) is sliding 1 m/s eastward (+) on a frictionless surface when it has an elastic collision with a 0.6 kg wood block (m_2) sliding 4 m/s westward (-). If (m_2) moves away at 2 m/s after the collision, what is the magnitude and direction of the block (m_1)?

sol; $p_{1i} + p_{2i} = p_{1f} + p_{2f}$ --------> $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

$v_{1f} = {}^{m_1v_{1i} + m_2v_{2i} - m_2v_{2f}}/_{m_1}$

$= {}^{(1.2\text{ kg})(1\text{ m/s}) + (0.6\text{ kg})(-4\text{ m/s}) - (0.6\text{ kg})(2\text{ m/s})}/_{1.2\text{ kg}}$

$= \mathbf{-2\ m/s}$ (westward)

When If the collision is inelastic the bodies will move off together with a common velocity and the above relation becomes $\mathbf{m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_f}$

Also keep in mind that (p) is a vector, so be aware of signs (+) (-) used.

1.0 N ~ 0.25 lb

ex; Two lumps of clay are thrown at each which collide head-on and stick together. One lump is 0.3 kg and travels to the right at 2.5 m/s, and the other lump is 0.5 kg and travels to the left at 4 m/s. What is the velocity and direction of the combined lump after the collision?

sol; Determine the (p) of each of the masses before the collision, choosing the (+) direction to be to the right.

$p_1 = (0.3 \text{ kg})(2.5 \text{ m/s}) = 0.75 \text{ kgm/s}$ & $p_2 = (0.5 \text{ kg})(-4 \text{ m/s}) = -2 \text{ kgm/s}$

- total (p) after collision = (+ 0.75 kgm/s) + (- 2 kgm/s) = -1.25 kgm/s
- combined mass of the lump of clay after collision = 0.8 kg
- (p) after collision = (0.8 kg)(v)

By equating the total (p) before and after the collision ----> -1.25 kgm/s = (0.8 kg)(v)

v = -1.5 m/s [The negative sign indicates that the combined lump is traveling to the left.]

ex; A 20,000 kg truck moving at 5 m/s collides into the rear of a 1,500 kg car which is at rest. The collision occurs just after the light turns green, and the driver of the car has their foot off the brake pedal. After this inelastic collision they move together as one in the same direction. At what speed will they be moving?

sol; $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

$m_1v_{1i} + 0 = (m_1 + m_2)v_f$

$v_f = \left(m_1v_{1i} / m_1 + m_2\right)$ ----> $v_f = (20{,}000 \text{ kg})(5 \text{ m/s}) / (20{,}000 \text{ kg} + 1{,}500 \text{ kg}) = \mathbf{4.6 \text{ m/s}}$

What can be said about the injuries each of the drivers might sustain from this collision?