

## Work & Energy

**work** ---> when an unbalanced force acts on an object through some distance

$$\mathbf{W} = \mathbf{F} \times \mathbf{d} \text{ [no motion = no work!]}$$

work ----> overcoming resistance

Note that work is a *scalar quantity* and has no direction associated with it, even though work depends on two vector quantities (force and displacement).

Two conditions must be met for work to have been done;

- 1). a force must be applied
- 2). body must move in direction of the (unbalanced) force.

MKS units:  $1.0 \text{ N} \times 1.0 \text{ m} = 1.0 \text{ Nm} = 1.0 \text{ Joule (J)}$

### *How do we determine the work done in lifting an object in the vertical direction?*

ex; A weightlifter pushes up a 300 N dumbbell for 0.75 meters. How much work is done on the dumbbell?

sol;  $W = 300 \text{ N} \times 0.75 \text{ m} = \mathbf{225 \text{ J}}$

ex; A 60 kg woman climbs up two flights of steps (~ 6.5 m). How much is done?

sol; What is the person doing work against? Her own weight! ---->  $W_t. = mg = (60 \text{ kg})(9.8 \text{ m/s}^2) = 588 \text{ N}$

How much work was done? ---->  $W = 588 \text{ N} \times 6.5 \text{ m} = \mathbf{3,822 \text{ J}}$

\*When lifting objects, if the direction is perpendicular to the earth's surface, use ---->  $\mathbf{W = mgh}$

-The work done in lifting mass vertically against gravity depends primarily on the vertical distance (h).

> In the previous examples, work was positive (+) since the unbalanced force and displacement were in the same direction. But when the displacement of a body is in a direction opposite to the unbalanced force we say that work is (-). This is the case when we consider work done against friction or gravity. For example;

- When a body is moved over a surface with friction, the displacement is in an opposite direction of friction.
- When a body is lifted up against the force of gravity, the displacement is in an opposite direction to the force. In this case, the work done raising a body up against gravity is stored up in the body as PE, and this work is conserved, that being the object can do work on the surface when it is released. This is known as a "conservative force."

ex; The work done compressing a spring is conservative because the PE of the spring has been increased.

- Forces that do not give rise to PE are called **non-conservative** forces, as the work done can not be returned by reversing the path. For example;

ex.'s; The work done (against frictional forces) when sliding a box across the floor is converted into heat, and is not available to move the box.

Work done against frictional forces always produces a rise in the temperature of the objects involved, and thus some of the work shows up as heat. ("non-conservative" force)

-When climbing a *spiral* staircase, what is the distance the force is applied through?

*At the earth's surface gravity works only in the vertical direction.*

## *Is it easier to climb stairs over a short or long period of time?*

An exertion of work over a shorter time period = an exertion of more power.

**power:** The rate at which work is done. How fast is work done?  $P = W/t = F \times d/t$

ex; 60 kg person climbing 6.5 meter stairs; 10 s vs. 30 s

$$\text{sol; } P_{10} = \frac{3,822 \text{ J}}{10 \text{ s}} \sim 382 \text{ J/s (382 W)}$$

$$P_{30} = \frac{3,822 \text{ J}}{30 \text{ s}} \sim 127 \text{ J/s (127 W)}$$

terminology - use “powerful” or “strong” force?

ex; What is the power output of an elevator motor which can lift a 1,200 kg car 25 meters in 18 seconds?

$$\text{sol; } P = W/t = mgh/t = 16,333 \text{ W (or 16.33 kW)}$$

\*Imperial units ----> ft-lb/s (550 ft-lb/s = 1.0 hp)

man ~ 1/2 hp (for a short time period!)
space shuttle ~ 1.2 x 10 <sup>7</sup> hp
Apollo XI Saturn V ~ 1.67 x 10 <sup>8</sup> hp

One horsepower is equal to lifting 550 pounds one foot in one second. 746 Watts = 1.0 horsepower A strong “dray” horse (clydesdale) is ~ 50% stronger than a race horse.
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**Energy:**

**energy:** (*Gk.* “ergon” = containing work) The ability to do work (or “the ability to cause change”).  
[units are the same as work!]

The term “energy” is coined by *Br.* Thomas Young in 1807.

> **Anything that can do work possesses energy, or if a body possesses energy it is capable of doing work.**

> **Anything which can cause an object to move is said to possess energy.**

ex; A compressed spring when released can do work on another body, and therefore it is said to have a store of energy. Energy, being a measure of the work it can do.

### **Energy is classified into three categories;**

**potential (PE):** [Concept developed by *Scot.* William Rankine in 1853.]

>The energy “stored” in an object due to its position in a field of force, or due to the distortion of its shape.

1. gravitational - “energy of position” depends on the height above ground [**PE = mgh**]
2. chemical - food, gunpowder, coal, petroleum, matches
3. elastic - compressed spring, stretched rubberband or spring, twisted matter (ex; sound)
4. electrical (separation) - “law of charges,” dust on tv screen, electrodes of battery
5. magnetic - attraction & repulsion due to polarity of a permanent magnet
6. phase - S>L>G
7. nuclear - split atom

**kinetic (KE):** [Term used by Br. Kelvin in 1856.]

>The energy that a “moving” object has (*or* “energy of motion”). [ $KE = \frac{1}{2}mv^2$ ]

\*The term “kinema” (later “cinema”) is the Greek word for “motion.”

1. mechanical - motion of any mass - matter

2. thermal - molecular motion

3. electric current - electrons in motion

4. radiant - electromagnetic energy [ex; light]

Heat engines convert thermal energy into mechanical energy.

\*When energy travels with the speed of light it is called “radiation.”

**Rest Energy:** The energy possessed in a body by virtue of its mass. Mass *is* a form of energy ---->  $E = mc^2$

Matter is not freely converted into energy, but *if all* intrinsic energy found in 1 kg of mass could be *completely* converted in to energy, then using the “mass-energy equivalence” relationship;

$E = mc^2$  ---->  $(1 \text{ kg})(3 \times 10^8 \text{ m/s})^2 = 9 \times 10^{16} \text{ J}$  [ $\sim 15,000,000$  barrels of crude oil.]

Transformation of Energy:

-Each form of energy can be changed directly (or indirectly) into any other form of energy.

[ex.'s; bow & arrow, swinging pendulum, rollercoaster, sawing & drilling wood, mousetrap, planetary motion, air resistance on a jet ---> where friction destroys mechanical motion and transforms it into heat]

>The “dissipation of (mechanical) energy” refers to the conversion into heat energy.

\* fuel ----> mechanical ----> electrical

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\* battery ----> electrical ----> mechanical

\* sun ----> plants ----> mechanical &/or heat

\* Heat can be generated from electrical energy (current) [ex.'s; heaters, toasters, etc..]

\***All forms of energy eventually end up as heat!**

Conservation of Energy: ( $\sim 1842$  Julius Mayer )

-Energy can be transferred from one body to another, but never destroyed; it is always conserved!

Individual values may change, but their sum does not; a falling ball always has same energy ( $\Delta KE = -\Delta PE$ ).

\*Show how a free-falling object’s PE (“falling force”) is converted into KE. Where did KE go?

[deformation of surfaces, sound, heat, light] \*head-on collision: where did KE go?

\*discuss pendulum motion; PE <---> KE conversion

**Conservation of energy is one of the most important of all the laws of nature!**

> In transforming energy from one form to another, energy is always conserved.

> Energy is never created or destroyed, only changed in form.

> The total sum of all the energy in the universe remains constant.

*Use  $\Delta PE = \Delta KE$  relationship to determine the velocity of a falling object.*

ex; How fast will a 1,000 kg rollercoaster be moving when it drops 40 meters? (Assume no friction!)

sol;  $-\Delta PE = \Delta KE$   $mgh = \frac{1}{2}mv^2$  ---->  $gh = \frac{v^2}{2}$  ---->  $2gh = v^2 = 784 \text{ m}^2/\text{s}^2$   $v = \sqrt{784 \text{ m}^2/\text{s}^2} = 28 \text{ m/s}$

ex; A 1,200 kg parked car rolls down a hill 14 meters high.

a) What is the kinetic energy of the car at the bottom? = **164,640 J**

b) What is the velocity of the car at the bottom? = **16.5 m/s** [curb wheels!]

ex; A 40 kg cannon ball is shot vertically upward at 80 m/s. According to the principle of conservation of energy, how high will the ball travel?

sol;  $\Delta PE = -\Delta KE$  ( $mgh = \frac{1}{2}mv^2$ ) *or* ( $gh = v^2/2$ ) .....  $h = v^2/2g = \mathbf{326.5\ m}$

Work - Energy Theorem: (Christiaan Huygens ~ 1673)

- The amount of work done on an object is equal to the energy the object possesses.

- In order to start or stop the motion of a body, work must be done on it.

- The KE given (or taken) from an object is equal to the work done on that object.

\* For example; When throwing a ball, work must be done on the ball. This amount of work shows up KE.

**$F \times d = \frac{1}{2}mv^2$**  (*or*  **$F \times d = mgh$**  for problems dealing with projectiles)

ex; How much work must be done to stop a 1,000 kg car moving at 10 m/s?

sol; How much KE does it have?  $KE = \frac{1}{2}mv^2 = 0.5 (1,000\ kg)(10\ m/s)^2 = 5 \times 10^4\ kgm^2/s^2 = \mathbf{5 \times 10^4\ J}$   
Therefore, according to this theorem;  $5 \times 10^4\ J$  of work must be done on the car to stop it.

ex; A 5 kg hammer strikes a nail at 3 m/s. If the distance the nail moved was 0.8 cm (0.008 m), what is the force on the nail?

sol;  $F \times d = \frac{1}{2}mv^2$  ---->  $F = 0.5 (5\ kg)(3\ m/s)^2 / 0.008\ m = 22.5\ kgm^2/s^2 / 0.008\ m = \mathbf{2,812.5\ N}$

historical note:

Credit for the discovery of conservation of energy is given to the German physician Robert Mayer. Having returned home to Germany in Feb. 1841 after a year on a Dutch East India vessel as a ship's doctor, he wrote down his thoughts on what he noticed when drawing blood from patients in the tropics. Mayer noticed that the blood color was much brighter ("uncommon redness") than in colder climates. Knowing what Lavoisier had stated in 1789 about the relationship about oxygen and heat during combustion, Mayer concludes that the brighter color of the blood was due to the lesser amount of oxidation required to keep up the body temperature in the tropics. From this he declares that the "*heat produced mechanically by an organism must bear an invariable quantitative relation to the work expended in producing it.*" This simple connection would lead him on to the discovery of "conservation of energy." In 1842 he publishes a paper entitled "*The Forces of Inorganic Nature.*" In it he discusses his thoughts on conservation of energy, and that "*a force once in existence cannot be annihilated.*" Aware of the various forms of forces (gravity, magnetic, electrical, and chemical), he saw each gave rise to the same thing ... "energy," and that all forms are mutually interchangeable. He went on to claim that whenever one of these kinds of energy disappears, one of the others always appears in its place. In machines, the loss of a given amount of work is only "apparent" and not real, as an exact equivalent of heat is produced. Another paper from him in 1845 entitled "*Organic Motion*" goes further stating how the sun pours out light energy which is taken up by plant life, which can be stored and used by animals for complex physiological processes, or stored in the earth in vast coal beds. The latter can be used as a fuel and transformed into heat and in turn steam, which could generate electricity.