New York City College of Technology
The City University of New York

Measurements and Density

Laboratory activity description
Physics Department
Physics PHYS 1433

Introduction.

Density characterizes how dense an object is. For example, it is clear that a pillow is less dense than a metal wrench. But how to characterize it in numbers?

How to find the density?

Find the mass of the object. This is done by weighing it. (We discuss the exact relationship between weight and mass later on, but one can use a weighing scale, without knowing this.) Whatever the value for the mass is, we will denote this value with the letter *m*. (Note: in some questions, the mass *m* might be given.)

1) *Find the volume of the object*. Well, it might be a little bit tricky if the object has a complicated shape. According to a legend, in ancient times Archimedes found the volume of a king's crown by measuring how much water was spilled from a big jar after he submerged the crown. In some cases, calculus can be used to find the volume - that is outside the scope of this algebra-based course. There are geometrical bodies, for which the volume formula is simple. For example, the volume *V* of a *box* is

$$V = l \times w \times h \qquad (1)$$

   where *l, w* and *h* are the length, width and height of the box. (Note: for different shapes, the formula for the volume is different. The formula above is only for a box. Some formulas for other shapes can be found here: https://en.wikipedia.org/wiki/Volume)

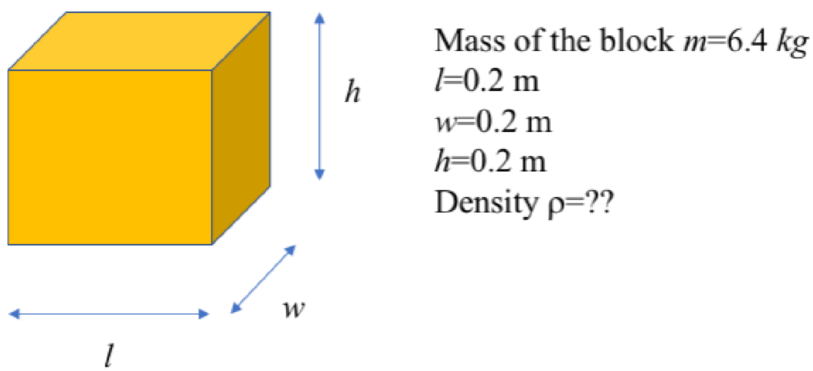
2) *Divide the mass m by the volume V.* This will be the numerical value of the density. (The Greek letter, ρ, pronounced "rho", is commonly used for the density.)

$$\rho = \frac{m}{V} \qquad (2)$$

Units of density can be constructed as a ratio of any mass and volume units. For example, if the mass is in pounds and volume is in cubic inches then the density is in pounds per cubic inch (lb/in^3). (Note this refers to the pound-mass i.e. the avoirdupois pound.)

In this physics course we mostly use SI units. In this system, the length is measured in meters (m) and the mass is measured in kilograms (kg). This is called the metric system or the International System. Thus, the unit of density is kilograms per cubic meter (**kg/m^3**).

Example. Let us suppose that we have a cubic wooden block of mass m=6.4 kg that has the length, width and height l=0.2 m, w=0.2 m, and h=0.2 m. Find the density of the block.



Solution: The mass is already given. The volume is not given but we can find it from the length, width and height of the block: The volume of the block is

$V = l*w*h$ = 0.2*0.2*0.2 = 0.008 m^3.

(Note: m^3 means cubic meters). Thus, the density of the block is its mass divided by its volume:

$$\rho = \frac{m}{V} = \frac{0.0064\ \text{kg}}{0.008\ \text{m}^3} = 0.8\ \frac{\text{kg}}{\text{m}^3}$$

Answer: the density of the wooden block is **800 kg/m^3**.

Uncertainty analysis. Our measurements cannot be perfectly accurate. In determining any quantity we have uncertainty. For example, standard rulers have millimeter markings. If we measure the length of an object as 20cm to the nearest millimeter, it might actually be 20.4 cm or 19.6 cm, just because that's the limitation of our grid and our eyesight. So, in this case the uncertainty $\Delta x = \pm 0.005\ cm$. It would also be reasonable to say the uncertainty is ±0.1 if we were not always confident about which marking was closer. So we can say that the length that we measure is $x = 20.0 \pm 0.1\ cm$

Another example – Below in this lab, we can weigh an object in our hand based on our everyday experience and say: Hey, this bag's weight is somewhat between 3 and 4 pounds (and I cannot tell more accurately). In this case, we can say that the weight is 3.5 ± 0.5 pounds. Here $\frac{3+4}{2} = 3.5$ pounds is the mean, and 0.5 is the uncertainty that gives us the boundaries: 3.5+0.5 = 4 pounds, the maximum value, and 3.5-0.5 gives us 3 pounds, the minimum value. And we expect that the actual weight is between these two boundaries.

What if a few uncertainties combine together? For example, if we know the value and uncertainty of the length, $l \pm \Delta l$, width, $w \pm \Delta w$, and height, $h \pm \Delta h$, of a box on Fig.1, what is the value and uncertainty of the box's volume? In science we talk about *propagation of uncertainty* in this case.

The answer is that, in this case, the volume *V* can be calculated via Equation (1) where l, *w* and *h* are the mean values. The *absolute* uncertainty (often referred to as just "uncertainty") of the volume can be estimated via the formula

$$\Delta V = V\left(\frac{\Delta l}{l} + \frac{\Delta w}{w} + \frac{\Delta h}{h}\right)$$

The *relative* uncertainty (or, *percent* uncertainty) is the uncertainty as a percentage of the measured or calculated value. For the volume calculation above, the *percent* uncertainty can be estimated as $\left(\frac{\Delta V}{V}\right) \times 100\%$.

Similarly, it is possible to find formulas for uncertainty of the density or, in general, uncertainty in any measurements or calculations. The formula for the percent uncertainty of density is

$$\left(\frac{\Delta l}{l} + \frac{\Delta w}{w} + \frac{\Delta h}{h} + \frac{\Delta m}{m}\right) \times 100\% = \frac{\Delta \rho}{\rho} \times 100\%$$

To get more information about uncertainty in measurements, and how to calculate it, watch this video https://youtu.be/_TqSI17EUXU or search in google or on YouTube "uncertainty in measurements".

Lab Assignment

1. Pick two objects to determine their density. It might be a pack of flour or a can. It can be anything that you find in your room. It is recommended that the objects have a marked weight. These are a couple of examples.

Fig.1 Example of two objects for the density determination (left) and their approximation as simple geometric shapes (right). In this lab, you use your own objects and their geometrical approximations. They do not have to be flour or a coffee can – just use what you find in your house.

2. Approximate the shapes of the chosen objects with simple geometrical shapes. For example, the shapes of the bad and a can could be approximated as a box and a cylinder, as shown in the figure above.

3. Measure relevant sizes of the objects to determine its volume. For example, in **Example** on page 2, for the box, the relevant sizes are the length, width and height. For the cylinder they are the diameter and the height, and so on. Measure the sizes in *centimeters* (cm) and in *inches* (in).

   If you do not have a ruler at home, this internet ruler is useful: https://www.ginifab.com/feeds/cm_to_inch/actual_size_ruler.html

4. Determine the volume of the two bodies you chose. You can find formulas for the volume of different shapes here: https://en.wikipedia.org/wiki/Volume

   Since you have already measured the sizes in *cm* and *in*, express the volumes in cm^3 and in in^3. Then convert the volume in *cubic centimeters* (cm^3) to *cubic meters* (m^3). If you have difficulty in conversion to *cubic meters*, google it or watch this (or similar) video on YouTube: https://youtu.be/YsMITlaHNSg

   At the end, you will have the volume of both objects in cubic meters (m^3) and in cubic inches (in^3).

5. Determine the mass of both objects. In the case shown in Fig. 1, the masses of a full bag and full can are labeled on the items. If your bag, for example, is half full, approximate your mass as one-half (1/2) of the labeled mass; it is only one-third full, approximate the mass as one-third (1/3) of the labeled mass.

   If you do not find at home objects with a labeled mass, weigh the objects in your hand. How many pounds do they weigh? Estimate the weight based on your everyday experience and write down your estimation. Convert this number to kilograms. You can google how to convert pounds to kilograms, if you have difficulty, or watch this 2 min video https://youtu.be/ilAxsmhxyGU

   At the end, you need the mass of the objects in both *kilograms* (*kg*) and *pounds (lb)*.

6. Determine the density of both objects in *kilograms per cubic meters* (kg/m^3) and in *pounds per cubic inch* (lb/in^3) from your measurements and Equation (2) above.

7. Organize your results as a data table. An example of the table is:

Object	Sizes in centi-meters (cm)	Volume in cubic centi-meter (cm^3)	Volume in cubic meters (m^3)	Sizes in in-ches (m)	Volume in cubic inches (in^3)	Mass in kilo-grams (kg)	Mass in pounds (lb)	Den-sity in kg/m^3	Den-sity in lb/in^3	Uncertainty %
Object 1: e.g., bag with flour	*List here all sizes inc m*	V==?? cm^3	V=?? m^3	*List here all sizes in in*	V=??	m=?? kg	m=?? in lb	ρ =?? kg/m^{33}	ρ =?? lb/in^{33}	$\Delta\rho$ =??%
Object 2,…										

This example of the table is just a guideline. You can organize your data as another table – feel free to come up with your own way or a table to organize your data. But at the end of the day, you want to get your data organized (this way or another) so that people, who read your report, understand you.

8. Estimate, what is the uncertainty of your measurements? For example, what is the uncertainty of your determination of length, volume and mass? Present your results in a format *mean* $\pm$ uncertainty (for example, $l \pm \Delta l$ for the length of the box, and so on).

   What is the resultant uncertainty of your determination of the density? Write down your finding and document the way how you get it. The best way is to add a picture of your calculations so that it is clear how you get the results.

   Estimate the absolute and relative uncertainties of the densities you determined.

   Question: Compare the obtained density of the objects with the density of water, 1000 kg/m^3. Are your objects denser or less dense than water? The rule (explained later in the course) is that if a body is denser than water, it sinks, and if a body is less dense than water it floats while it is put in water. Do you think your objects will sink or float? Is your expectation supported by the comparison of the density you determined with the density of water? Note: You do not need to actually put the objects in the water – the water can damage it. Just explain in writing what you think.

9. Your report should contain your table, the answer to the question above, explanation of equipment used to measure(e.g. ruler, internet ruler, scale, guesstimate, etc.), calculation of volume, density, and uncertainty. If you have a smartphone, or a camera, take pictures of measured objects and add the picture to your report. If not, describe your objects in words and/or sketch and/or find an image online.

Adapted from Activity Designed by Dr. German Kolmakov and Shaina Raklyar